1. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

a) The position vectors are scalar multiples of each other

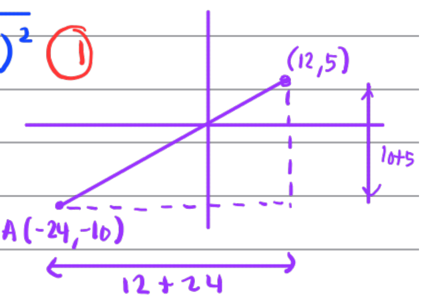
$(-24\underline{i} - 10\underline{j}) = -2(12\underline{i} + 5\underline{j})$ (1)

$\overrightarrow{AO} = -2(\overrightarrow{OB})$

Hence, the vectors $\overrightarrow{AO}$ and $\overrightarrow{OB}$ are parallel, and as the stone is travelling in a straight line $\overrightarrow{AB}$, the stone passes through the point O as $\overrightarrow{AB}$ does. (1)

b) The distance AB $= \sqrt{(12+24)^2 + (10+5)^2}$ (1)

$= 39$ m



The speed of the stone $= \dfrac{39}{4}$ (1) $= 9.75$ m/s (1)

2. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

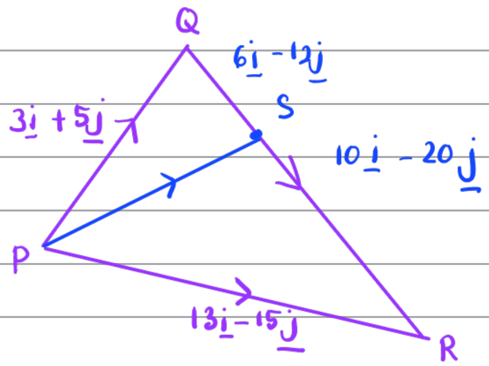
(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

(c) Find $\overrightarrow{PS}$

(2)



a) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR} = -(3\underline{i} + 5\underline{j}) + (13\underline{i} - 15\underline{j})$ (1)

$= 10\underline{i} - 20\underline{j}$ (1)

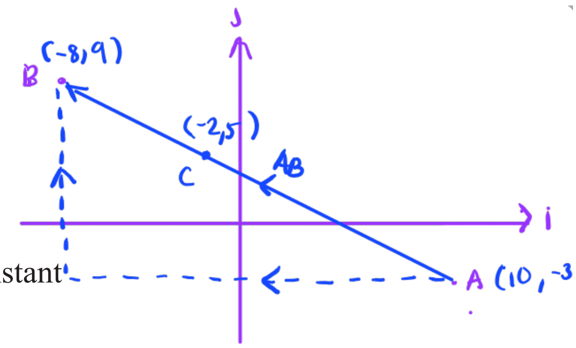
b) $|\overrightarrow{QR}| = \sqrt{10^2 + (-20)^2}$ (1)

$= 10\sqrt{5}$ (1)

c) $\overrightarrow{QS} = \frac{3}{5}\overrightarrow{QR} = \frac{3}{5}(10\underline{i} - 20\underline{j}) = 6\underline{i} - 12\underline{j}$ (1)

$\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS} = (3\underline{i} + 5\underline{j}) + (6\underline{i} - 12\underline{j})$ (1)

$= 9\underline{i} - 7\underline{j}$ (1)



3. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + {}^{5}p\mathbf{j}$ where p is a constant

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

$\frac{12}{18} \cdot \frac{2}{3}$

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(10\underline{i} - 3\underline{j}) + (-8\underline{i} + 9\underline{j})$ (1)

$= -10\underline{i} - 8\underline{i} + 3\underline{j} + 9\underline{j}$

$= -18\underline{i} + 12\underline{j}$ (1)

b) $\left|\overrightarrow{AB}\right| = \sqrt{(-18)^2 + (12)^2}$

$= \sqrt{468}$ (1)

$= \sqrt{36} \times \sqrt{13}$

$= 6\sqrt{13}$ (1)

(c) (i) gradient BC = gradient BA (because all points are on the same line)

$m_{BC} = \frac{9-p}{(-8)-(-2)} = \frac{9-p}{-6}$ ①

$m_{BA} = \frac{9-(-3)}{(-8)-10} = \frac{12}{-18}$

so, $\frac{9-p}{-6} = \frac{12}{-18}$

$3(9-p) = 12$

$27-3p = 12$

$3p = 15$

$p = 5$ ①

(ii) since length AC : AB is 2 : 3,

ratio of triangle AOC is 2:3 to triangle AOB. ①

4.

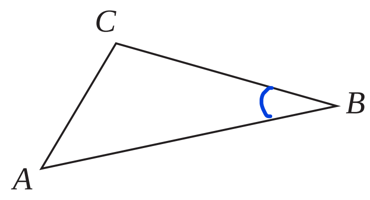


Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ 1

$\overrightarrow{AC} = -2\underline{i} - 3\underline{j} - \underline{k}$ 1

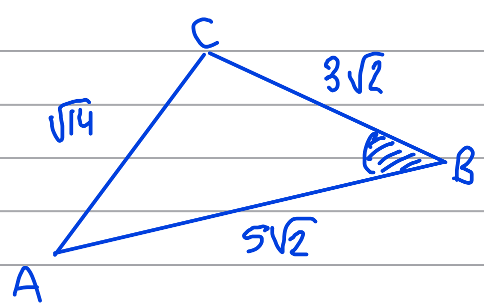
b) $|AC| = \sqrt{(-2)^2 + (-3)^2 + (-1)^2} = \sqrt{14}$

cosine rule :

$|AB| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2} = 5\sqrt{2}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$|BC| = \sqrt{1^2 + 1^2 + 4^2} = 3\sqrt{2}$ 1



$\cos ABC = \dfrac{(5\sqrt{2})^2 + (3\sqrt{2})^2 - (\sqrt{14})^2}{2 \times 5\sqrt{2} \times 3\sqrt{2}}$ 1

$\cos ABC = \dfrac{9}{10}$ 1

5.

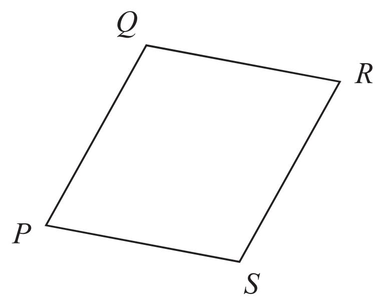


Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ ← bold letters represent vectors
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram $PQRS$ is a rhombus. ← all 4 sides are the same length **(2)**

(b) Find the exact area of the rhombus $PQRS$. **(4)**

(a) $|\overrightarrow{PQ}| = \sqrt{2^2 + 3^2 + (-4)^2}$ ← $|v|$ is the magnitude (length) of v.

$= \sqrt{29}$

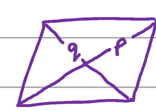
$|\overrightarrow{QR}| = \sqrt{5^2 + (-2)^2}$ (1)

$= \sqrt{29}$

Since we know PQRS is a parallelagram, we only need to calculate the length of 2 of the 4 sides

$|\overrightarrow{PQ}| = |\overrightarrow{QR}|$ $\therefore$ PQRS is a rhombus. (1)

area of a rhombus $= \frac{p \times q}{2}$



(b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= (2i + 3j - 4k) + (5i - 2k)$

$= 7i + 3j - 6k$ (1)

$\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS}$ ← we're going 'backwards' along PQ.

$= -(2i + 3j - 4k) + (5i - 2k)$ ← $\overrightarrow{PS} = \overrightarrow{QR}$

$= 3i - 3j + 2k$ (1)

$$\text{Area} = \frac{|\overrightarrow{PR}| \times |\overrightarrow{QS}|}{2} \quad (1)$$

$$= \frac{\sqrt{7^2 + 3^2 + (-6)^2} \times \sqrt{3^2 + (-3)^2 + 2^2}}{2}$$

$$\therefore \text{Area} = \sqrt{517} \quad (1)$$

6. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\overrightarrow{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

a) $|\overrightarrow{OA}| = \sqrt{5^2+3^2+2^2} = \sqrt{38}$ ①

b) $|\overrightarrow{OB}| = \sqrt{2^2+4^2+a^2} = \sqrt{20+a^2}$

when $a=4$, $|\overrightarrow{OB}| = \sqrt{36} < \sqrt{38}$

when $a=5$, $|\overrightarrow{OB}| = \sqrt{45} > \sqrt{38}$ ①

$\therefore a = 5$ ①

7. Relative to a fixed origin O,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC.

(2)

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

a) Method: find $\overrightarrow{AD}$ and $\overrightarrow{BC}$ and show that one is a multiple of the other

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = -12\underline{i} + (22\underline{i} + 24\underline{j}) = 10\underline{i} + 24\underline{j}$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -16\underline{j} + (50\underline{i} + 136\underline{j}) = 50\underline{i} + 120\underline{j}$ (1)

$= 5\overrightarrow{AD}$ (1)

$\therefore$ AD is parallel to BC

b) average speed $= \dfrac{\text{total distance}}{\text{total time}}$

total time = 5 minutes = $\dfrac{5}{60}$ hours

total distance $= 2(|AB| + |BC| + |CD| + |DA|)$

$AB = \overrightarrow{AO} + \overrightarrow{OB} = -12\underline{i} + 16\underline{j}$

$CD = \overrightarrow{CO} + \overrightarrow{OD} = -(50\underline{i} + 136\underline{j}) + (22\underline{i} + 24\underline{j}) = -28\underline{i} - 112\underline{j}$

$|AB| = \sqrt{12^2+16^2} = 20\text{ m}$

$|BC| = \sqrt{50^2+120^2} = 130\text{ m}$ (1)

$|CD| = \sqrt{28^2+112^2} = 28\sqrt{17}\text{ m}$

$|DA| = \sqrt{10^2+24^2} = 26\text{ m}$ (1)

distance of 2 laps $= 2(176 + 28\sqrt{17})\text{ m}$

$= \dfrac{2(176+28\sqrt{17})}{1000}\text{ km}$

speed $= \dfrac{[2(176+28\sqrt{17})/1000]\text{ km}}{[5/60]\text{ h}}$ (1) $= 6.99\text{ kmh}^{-1}$ (3sf) (1)